Properties of Rational Exponents

Goals
- Use properties of rational exponents.
- Use properties of rational exponents in real life.

VOCABULARY

Simplest form of a radical: A radical expression after you apply the properties of radicals, remove any perfect nth powers, and rationalize any denominators.

Like radicals: Two radical expressions that have the same index and the same radicand.

PROPERTIES OF RATIONAL EXPONENTS

Let $a$ and $b$ be real numbers and let $m$ and $n$ be rational numbers. The following properties have the same names as those listed in Lesson 6.1, but now apply to rational exponents as illustrated.

<table>
<thead>
<tr>
<th>Property</th>
<th>Example</th>
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</thead>
<tbody>
<tr>
<td>1. $a^m \cdot a^n = a^{m+n}$</td>
<td>$3^{1/2} \cdot 3^{3/2} = 3^{(1/2 + 3/2)} = 3^2 = 9$</td>
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<tr>
<td>2. $(a^m)^n = a^{mn}$</td>
<td>$(4^{3/2})^2 = 4^{(3/2 \cdot 2)} = 4^3 = 64$</td>
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<tr>
<td>3. $(ab)^m = a^m b^m$</td>
<td>$(9 \cdot 4)^{1/2} = 9^{1/2} \cdot 4^{1/2} = 3 \cdot 2 = 6$</td>
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<tr>
<td>4. $a^{-m} = \frac{1}{a^m}, a \neq 0$</td>
<td>$25^{-1/2} = \frac{1}{25^{1/2}} = \frac{1}{5}$</td>
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<tr>
<td>5. $\frac{a^m}{a^n} = a^{m-n}, a \neq 0$</td>
<td>$\frac{6^{5/2}}{6^{1/2}} = 6^{(5/2 - 1/2)} = 6^2 = 36$</td>
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<tr>
<td>6. $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}, b \neq 0$</td>
<td>$\left(\frac{8}{27}\right)^{1/3} = \frac{8^{1/3}}{27^{1/3}} = \frac{2}{3}$</td>
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Your Notes

Example 1  Using Properties of Rational Exponents

Use properties of rational exponents to simplify the expression.

a. \(7^{2/3} \cdot 7^{1/2} = 7^{(2/3 + 1/2)} = 7^{7/6}\)

b. \((3^{1/4} \cdot 5^{1/3})^3 = (3^{1/4})^3 \cdot (5^{1/3})^3 = (3^{1/4} \cdot 3) \cdot (5^{1/3} \cdot 3) = 3^{3/4} \cdot 5\)

c. \(\frac{10^{1/2}}{10^{1/4}} = 10^{(1/2 - 1/4)} = 10^{1/4}\)

d. \(\left(\frac{12^{1/6}}{3^{1/6}}\right)^4 = \left[\left(\frac{12}{3}\right)^{1/6}\right]^4 = (4^{1/6})^4 = 4^{(1/6 \cdot 4)} = 4^{2/3}\)

Example 2  Using Properties of Radicals

Use the properties of radicals to simplify the expression.

a. \(\sqrt[5]{16} \cdot \sqrt[5]{2} = \sqrt[5]{16 \cdot 2} = \sqrt[5]{32} = 2\)  Use the product property.

b. \(\frac{\sqrt[3]{108}}{\sqrt[3]{4}} = \sqrt[3]{\frac{108}{4}} = \sqrt[3]{27} = 3\)  Use the quotient property.

Checkpoint  Simplify the expression.

1. \((6^5 \cdot 2^5)^{-1/5}\)  2. \(\frac{\sqrt[4]{48}}{\sqrt[4]{3}}\)

\[\frac{1}{12}\] \[2\]

Example 3  Writing Radicals in Simplest Form

Write the expression in simplest form.

\[\frac{\sqrt[4]{48}}{\sqrt[4]{3}} = \frac{\sqrt[4]{16 \cdot 3}}{\sqrt[4]{3}}\]  Factor out perfect fourth power.

\[= \frac{\sqrt[4]{16}}{\sqrt[4]{3}} \cdot \frac{\sqrt[4]{3}}{\sqrt[4]{3}}\]  Product property

\[= \frac{2\sqrt[4]{3}}{\sqrt[4]{3}}\]  Simplify.
Example 4  Adding and Subtracting Roots and Radicals

Perform the indicated operation.

a. \(3\left(5^{3/4}\right) + 4\left(5^{3/4}\right) = \left(3 + 4\right)\left(5^{3/4}\right) = 7\left(5^{3/4}\right)\)

b. \(\sqrt[4]{64} + \sqrt[4]{4} = \sqrt[4]{16 \cdot 4} + \sqrt[4]{4} = 2\sqrt[4]{4} + \sqrt[4]{4} = (2 + 1)\sqrt[4]{4} = 3\sqrt[4]{4}\)

Checkpoint Write the expression in simplest form.

3. \(\sqrt[4]{\frac{2}{9}}\)  

4. \(\sqrt[4]{64} - \sqrt[4]{4}\)  

\(\frac{\sqrt[4]{18}}{3}\)  

\(\sqrt[4]{4}\)

Example 5  Simplify Expressions Involving Variables

Simplify the expression. Assume all variables are positive.

a. \(\sqrt[4]{81x^{12}} = \sqrt[4]{3^4(x^3)^4} = 3x^3\)

b. \((8a^9b^{3})^{1/3} = 8^{1/3}(a^9)^{1/3}(b^3)^{1/3} = 2a^{(9 \cdot 1/3)}b^{(3 \cdot 1/3)} = 2a^3b\)

c. \(\sqrt[6]{\frac{y^{12}}{x^{24}}} = \frac{\sqrt[6]{y^{12}}}{\sqrt[6]{x^{24}}} = \frac{\sqrt[6]{(y^2)^6}}{\sqrt[6]{(x^4)^6}} = \frac{y^2}{x^4}\)

d. \(\frac{8x^2y^6z^2}{4x^{1/4}y^5} = 2x^{(2 - 1/4)}y^{(6 - 5)}z^2 = 2x^{7/4}yz^2\)
**Example 6**  *Writing Variable Expressions in Simplest Form*

Write the expression in simplest form. Assume all variables are positive.

\[
\frac{\sqrt[3]{y^2}}{\sqrt[5]{x^5}} = \frac{\sqrt[3]{y^2x}}{\sqrt[5]{x^5x}}
\]

Make the denominator a perfect cube.

\[
\frac{\sqrt[3]{y^2x}}{\sqrt[6]{x^6}} = \frac{\sqrt[3]{y^2x}}{\sqrt[6]{x^6}}
\]

Simplify.

\[
\frac{\sqrt[3]{y^2x}}{\sqrt[6]{x^6}} = \frac{\sqrt[3]{y^2x}}{\sqrt[6]{x^6}}
\]

Quotient property

\[
\frac{\sqrt[3]{y^2x}}{\sqrt[6]{x^6}} = \frac{\sqrt[3]{y^2x}}{\sqrt[6]{x^6}}
\]

Simplify.

**Example 7**  *Adding and Subtracting Variable Expressions*

Perform the indicated operation. Assume all variables are positive.

a. \(3\sqrt[3]{x} - \sqrt[3]{x} = (3 - 1)\sqrt[3]{x} = 2\sqrt[3]{x}\)

b. \(5x^2y^{1/2} + 7x^2y^{1/2} = (5 + 7)x^2y^{1/2} = 12x^2y^{1/2}\)

**Checkpoint** Simplify the expression. Assume all variables are positive.

5. \(\frac{14xy^3z^5}{2x^{2/3}yz^2}\)

6. \(4\sqrt[3]{5y^4} - y\sqrt[3]{40y}\)

7. \(7x^{1/3}y^{2/3}z^3\)

8. \(2y\sqrt[3]{5y}\)