

10.1

The Distance and Midpoint Formulas

- Goals**
- Use the distance and midpoint formulas.
 - Use the distance and midpoint formulas in real-life situations.

Your Notes

THE DISTANCE FORMULA

The distance d between the points (x_1, y_1) and (x_2, y_2) is as follows: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Example 1 Finding the Distance Between Two Points

Find the distance between $(4, -5)$ and $(-2, 7)$.

Let $(x_1, y_1) = (4, -5)$ and $(x_2, y_2) = (-2, 7)$.

$$\begin{aligned}d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} && \text{Use distance formula.} \\&= \sqrt{(-2 - 4)^2 + (7 - (-5))^2} && \text{Substitute.} \\&= \sqrt{36 + 144} && \text{Simplify.} \\&= \sqrt{180} \approx 13.42 && \text{Use a calculator.}\end{aligned}$$

Example 2 Classifying a Triangle Using the Distance Formula

Classify $\triangle ABC$ as scalene, isosceles, or equilateral.

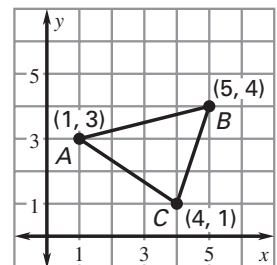
Solution

$$AB = \sqrt{(5 - 1)^2 + (4 - 3)^2} = \sqrt{17}$$

$$BC = \sqrt{(5 - 4)^2 + (4 - 1)^2} = \sqrt{10}$$

$$AC = \sqrt{(4 - 1)^2 + (1 - 3)^2} = \sqrt{13}$$

Triangle ABC is scalene.



✓ **Checkpoint** Complete the following exercises.

1. Find the distance between the points $(0, 2)$ and $(-4, 5)$.

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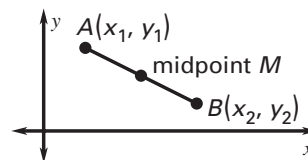
2. The vertices of a triangle are $(1, 0)$, $(1, 6)$, and $(3, 3)$.
Classify the triangle as *scalene*, *isosceles*, or *equilateral*.

isosceles

THE MIDPOINT FORMULA

The midpoint of the line segment joining $A(x_1, y_1)$ and $B(x_2, y_2)$ is as follows:

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$



Each coordinate of M is the mean of the corresponding coordinates A and B .

Example 3 Finding the Midpoint of a Segment

Find the midpoint of the line segment joining $(1, -4)$ and $(-3, 5)$.

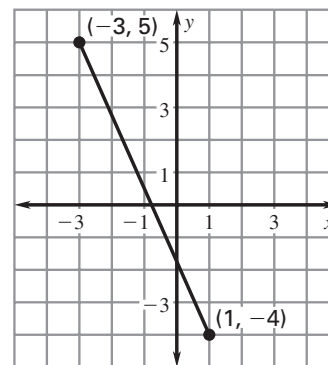
Solution

Let $(x_1, y_1) = (1, -4)$ and

$(x_2, y_2) = (-3, 5)$.

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$= \left(\frac{1 + (-3)}{2}, \frac{-4 + 5}{2}\right) = \left(-1, \frac{1}{2}\right)$$



Example 4 Finding a Perpendicular Bisector

Write an equation for the perpendicular bisector of the line segment joining $A(-2, 3)$ and $B(2, 5)$.

Solution

First find the midpoint of the line segment:

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left(\frac{-2 + 2}{2}, \frac{3 + 5}{2} \right) = (0, 4)$$

Then find the slope of \overline{AB} :

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 3}{2 - (-2)} = \frac{2}{4} = \frac{1}{2}$$

The slope of the perpendicular bisector is the negative reciprocal of $\frac{1}{2}$, or $m_{\perp} = \underline{-2}$.

Because you know the slope of the perpendicular bisector and a point that the bisector passes through, you can use the point-slope form to write its equation.

$$\begin{aligned} y - 4 &= -2(x - 0) \\ y &= \underline{-2x + 4} \end{aligned}$$

✔ **Checkpoint** Complete the following exercises.

3. Find the midpoint of the line segment joining $(-2, -1)$ and $(2, 3)$.

$$(0, 1)$$

4. Write an equation for the perpendicular bisector of the line segment joining $A(-1, -3)$ and $B(3, 7)$.

$$y = -\frac{2}{5}x + \frac{12}{5}$$

Homework